



$$\nabla^2 \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla p}{\rho} + \frac{\mu}{\rho} \nabla^2 \vec{v} + \vec{g}$$

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + g_z$$

$$\frac{\partial}{\partial z} = 0, \quad \frac{\partial}{\partial r} = 0, \quad \frac{\partial}{\partial \theta} = 0, \quad \frac{\partial}{\partial t} = 0$$

$$v_\theta = v_r = 0$$

$$\frac{dp}{dz} = \frac{\mu}{r} \frac{d}{dr} \left(r \frac{dv_z}{dr} \right)$$

v_z é finita em $r=0$

$v_z = 0$ em $r=R$

$$\frac{d}{dr} \left(r \frac{dv_z}{dr} \right) = \frac{r}{\mu} \frac{dp}{dz}$$

$$r \frac{dv_z}{dr} = \frac{r^2}{2\mu} \frac{dp}{dz} + C_1$$

$$v_z(r) = \frac{r^2}{4\mu} \frac{dp}{dz} + C_2$$

$$v_z(R) = \frac{R^2}{4\mu} \frac{dp}{dz} + C_2 = 0$$

$$v_z(r) = \frac{r^2}{4\mu} \frac{dp}{dz} - \frac{R^2}{4\mu} \frac{dp}{dz} = \frac{R^2}{4\mu} \left(-\frac{dp}{dz} \right) \left(1 - \frac{r^2}{R^2} \right)$$

$$v_{\max} = v_z(0) = \frac{R^2}{4\mu} \left(-\frac{dp}{dz} \right)$$

$$v_m = \frac{Q}{A} = \frac{\int v_z dA}{A} = \frac{\int_0^R \int_0^{2\pi} v_z(r) r d\theta dr}{\pi R^2}$$

$$= \frac{\int_0^R v_z(r) 2\pi r dr}{\pi R^2} = \frac{\left(-\frac{dp}{dz} \right) \frac{R^2}{4\mu}}{\pi R^2} \int_0^R \left(1 - \left(\frac{r}{R} \right)^2 \right) 2\pi r dr$$

$$= \left(-\frac{dp}{dz} \right) \frac{1}{2\mu} \int_0^R \left(r - \frac{r^3}{R^2} \right) dr = \left(-\frac{dp}{dz} \right) \frac{1}{2\mu} \left[\frac{R^2}{2} - \frac{R^4}{4R^2} \right] = \left(-\frac{dp}{dz} \right) \frac{R^2}{8\mu}$$

$$\boxed{\left(-\frac{dp}{dz} \right) = \frac{8\mu v_m}{R^2}}$$